CSISS WORKSHOP

Introduction to Spatial Pattern Analysis in a GIS Environment

Geostatistics: The Semivariogram and Kriging

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Geostatistics

- Semivariance and the Semivariogram
  (also semivariogram = variogram)

- Kriging

Semivariance

- A measure of the degree of spatial dependence between observations of a regionalized variable.
- Formulation

$$\gamma_h = \Sigma (x_i - x_{i+h})^2 / 2n$$

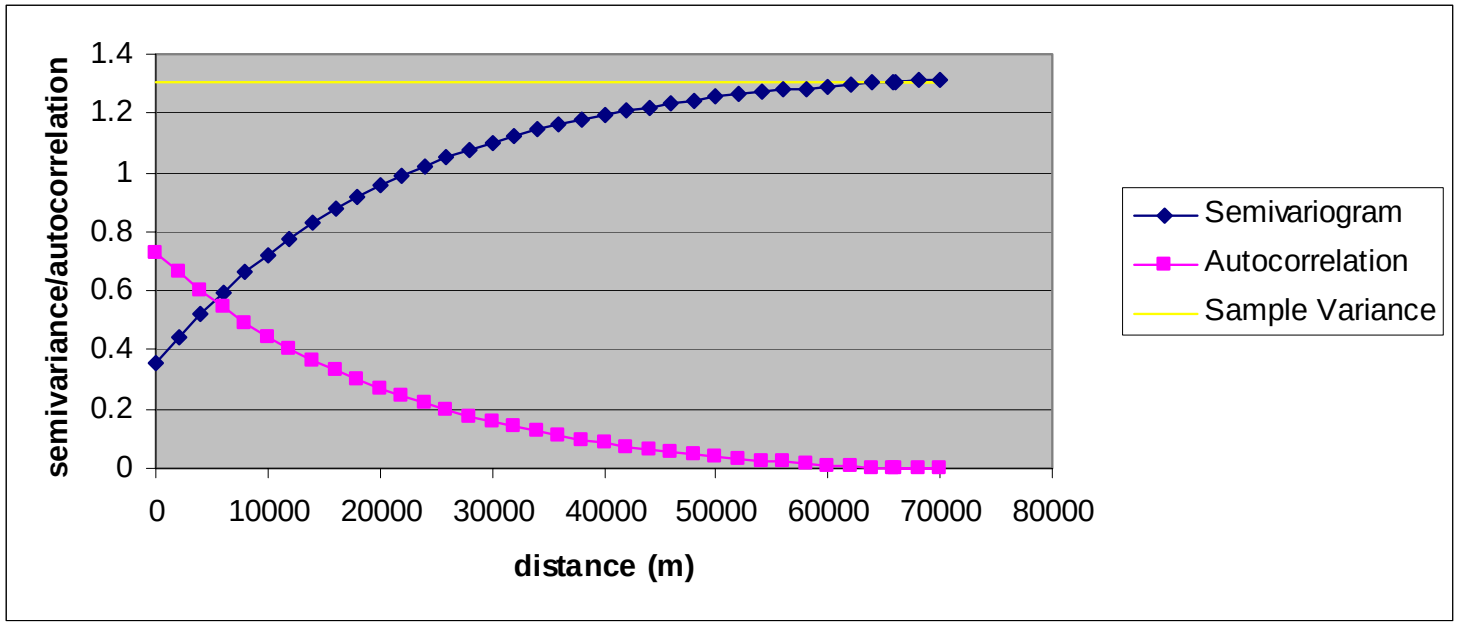
where h is the distance interval between points.

The plot for a number of h's is called the semivariogram.

Intrinsic Stationarity

Variogram analysis cannot proceed without acceptable assumptions, chief of which is *intrinsic stationarity*.

Geostatistics Model



$$\gamma(d) = 0.356 + \left[1 - e^{\frac{-3d}{65700}}\right]$$

$$W_{ij} = 1 - \left(\gamma(d_{ij}) / \sigma^2\right)$$

Characteristics of Semivariogram

- Range
- Sill
- Nugget
- Autocorrelation
- Variance = Sill

The Geostatistics Models

Assumptions and Definitions

- Intrinsic Stationarity: $2\gamma(d) = \text{Var}\,[Y(u) - Y(v)]$
- The Semivariance: $\gamma(d) = [1/2N] \; \Sigma \{Y(u) - Y(v)\}^2$
- Autocorrelation: $\rho(d) = 1 - [\gamma(d) / \sigma^2]$
- The Range: when $\gamma = \sigma^2$
- The Nugget: when $\gamma > 0$ at $d = 0$

Semivariograms

- OBSERVED
- THEORETICAL
  - Spherical
  - Exponential
  - Linear (with sill)
  - Gaussian

The Geostatistics Models

- Spherical

$$w_{ij} = 1 - \sigma^2 \left(\frac{3d_{ij}}{2d_r} - \frac{d_{ij}^3}{2d_r^3} \right) \qquad d_{ij} \le d_r$$

- Gaussian

$$w_{ij} = 1 - \sigma^2 \left(1 - e^{-3d_{ij}^2 / d_r^2} \right) \qquad d_{ij} \le d_r$$

- Exponential

$$w_{ij} = 1 - \sigma^2 \left(1 - e^{-3d_{ij} / d_r} \right) \qquad d_{ij} \le d_r$$

Kriging

- The Idea of Kriging

- Models
  - Simple (punctual)
  - Ordinary (punctual)
  - Universal (punctual)
  - Block
  - Cokriging
  - Others

Simple Kriging

- $Z(x_0) = m + \mathbf{YW^{-1}B}$
- where m = assumed mean (known)
- **Y** = observations in the vicinity of x_0 (-m)
- **W** = correlation - semivariance (for all pairs of observations)
- **B** = correlation - semivariance (for all pairs between observations and x_0)

Ordinary Kriging

- $Z(x_0) = \mathbf{YW^{-1}B}$

Universal Kriging

- Drift

Block Kriging

- Areas or volumes

Cokriging

- More than one variable used to estimate value at a particular location.