CSISS WORKSHOP

Introduction to Spatial Pattern Analysis in a GIS Environment

Spatial Modeling: Spatial Parameters in Autoregressive Models and
Geographically Weighted Regression

Arthur Getis

Typology of Spatial Econometric Models

- General Model:

$Y = \theta W_1 Y + X\beta + \varepsilon$

$\varepsilon = \kappa W_2 \varepsilon + \lambda$ with λ normal, 0 mean, and constant variance Ω (i.e., variance is the same for every variable and covariance for every combination of variables is always 0)

Typology of Spatial Econometric Models

$Y = \theta W_1 Y + X\beta + \varepsilon$ and $\varepsilon = \kappa W_2 \varepsilon + \lambda$

set $\theta=0, \kappa = 0$ ------ RESULT is $Y = X\beta + \varepsilon$

set $\kappa = 0$ ------- RESULT is $Y = \theta W_1 Y + X\beta + \varepsilon$

set $\theta=0$ ------- RESULT is $Y = X\beta + (I - \kappa W_2)^{-1} \lambda$

also ------- $Y = \theta W_1 Y + X\beta + (I - \kappa W_2)^{-1} \lambda$

These are: linear regression model

spatial lag model

spatial disturbance model

spatial lag and disturbance model

Typology of Spatial Econometric Models

- Also, vary variance and covariance assumptions. (Ω can represent heteroscedasticity as well as homoscedasticity)
- Also, can create space-time models

Testing the Models

- In any model, if error term is correlated OLS is inappropriate device to find parameters. Usual tests on parameters and R^2 cannot be used.
- Use maximum likelihood approach (i.e., the parameters most likely to give you your data).

Wald test on parameters; Likelihood Ratio test on the goodness of the model; **La Grange Multiplier** test on residuals (non-spatial); **Moran's I** test on residuals (spatial)

Example

- Crime in Columbus
- Bisection search for θ parameter yields 0.4310.

  $CR = 45.1 + 0.43\ W_1CR - 1.0\ IN - 0.3\ Ho$
- Now, La Grange Test on Residuals (non-spatial) = OK;

  $Z(I) = 0.65$
- Conclusion: Not strong evidence of a spatial disturbance process after introduction of a spatially lagged dependent variable.

Geographically Weighted Regression (GWR)
"Geography of Parameter Space"

- Based on need to deal with heterogeneity in geographical space.
- Y_i is explained by X_j within *d*.
- One equation for each Y_i.
- X_j are weighted by distance from *i*.
- *d* represents a kernel size.
- Repeat for all *i*.
- $Y_i = a_i + b_i \sum X_j[d/w] + e$ *for all j within d of i; w is a spatial weight*

Problems Associated with Spatial Pattern Analysis

The Problems Help to Define the Field

- Scale (statistics that vary *d* -- LISA, G, K, ESDA)
- Zoning (use smallest units and then aggregate upward if necessary; use statistics that vary *d*; ESDA)
- **Dependence** (SpaStats, Variogram, Spa. Econ.)
- Heterogeneity (statistics that vary *d*, GWR, Spa. Econ.)
- Boundaries (NN, K function)
- Missing Data (Kriging, TINs)
- Large Data Sets (all of the above)

S p a t i a l is S p e c i a l